Brownian Motion near a Soft Surface

Yilin YE[1,2]
Yacine AMAROUCHENE[1], David DEAN[1], Thomas SALEZ[1]

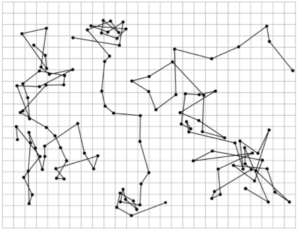
[1]Laboratoire Ondes et Matière d'Aquitaine, Univ. Bordeaux
[2]École Normale Supérieure, Université Paris Sciences et Lettres

Journées de la matière condensée 2022 à Lyon, France
22 - 26 août
Société Française de Physique

MC24 - Simulations des colloïdes

◦ Brownian motion

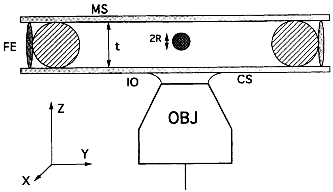
- 1827, Robert Brown: random motion of pollen particles;
- 1901, Louis Bachelier: theory of speculation;
- 1905, Albert Einstein: diffusive model; $D_{\mathrm{bulk}} = \frac{k_{\mathrm{B}} T}{6\pi\eta r}$
- 1908, Paul Langevin: equations of motions;
- 1909, Jean Perrin: experiments to measure N_{A};

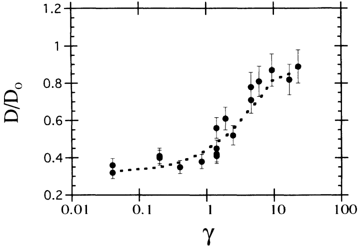


Brownian motion of pollen
J. Perrin, in *Atoms*, London: Constable **1914**, pp. 115.

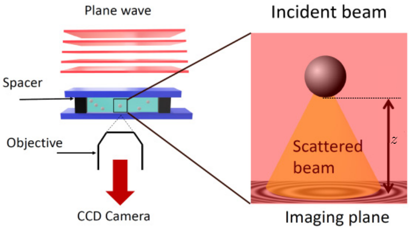
Brownian motion of stock price

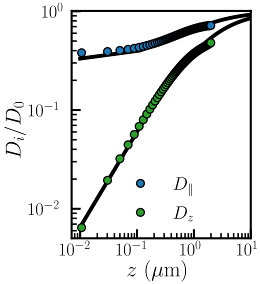
◦ Confined Brownian motion





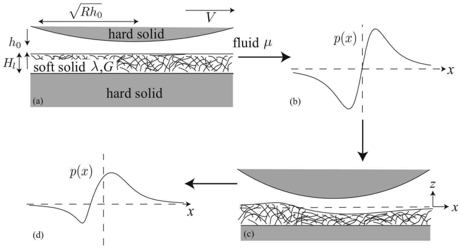
L. Faucheux, A. Libchaber. *Phys. Rev. E.* **1994**, *49(6)*, 5158.



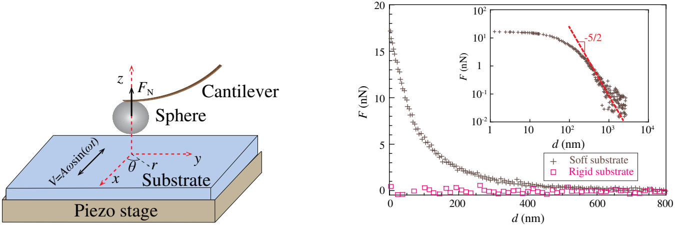


M. Lavaud, et al. *Phys. Rev. Res.* **2021**, *3(3)*, L032011.

◦ ElastoHydroDynamic lift force



J. Skotheim, L. Mahadevan. *Phys. Rev. Lett.* **2004**, *92*, 245509.



Z. Zhang, et al. *Phys. Rev. Lett.* **2020**, *124(5)*, 054502.

⊙ Situation of the problem

Brownian motion Confined ElastoHydroDynamics

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Brownian motion Confined ElastoHydroDynamics

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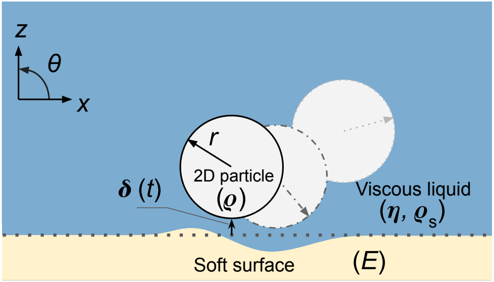
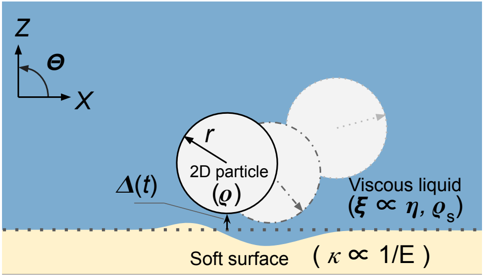


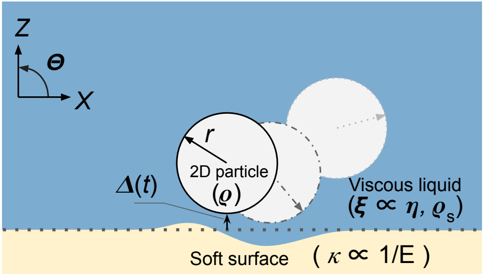
- $\xi = \frac{3\sqrt{2}\eta}{r^{3/2}\varepsilon\sqrt{\rho(\rho-\rho_s)g}}$ dimensionless viscosity;
- $\kappa = \frac{2h_s\eta\sqrt{g(\rho-\rho_s)}}{r^{3/2}\varepsilon^{5/2}(2\mu+\lambda)\sqrt{\rho}}$ dimensionless compliance; inverse of Young's modulus;

⊙ Elastohydrodynamic interactions



$$x = X_G \cdot r\sqrt{2\varepsilon}$$
$$\delta = \Delta \cdot r\varepsilon$$
$$\theta = \Theta \cdot \sqrt{2\varepsilon}$$
$$t = T \cdot r\sqrt{2\varepsilon}$$

⊙ Elastohydrodynamic interactions



$$x = X_{\mathrm{G}} \cdot r\sqrt{2\varepsilon}$$

$$\delta = \Delta \cdot r\varepsilon$$

$$\theta = \Theta \cdot \sqrt{2\varepsilon}$$

$$t = T \cdot r\sqrt{2\varepsilon}$$

$$\ddot{X}_{\mathrm{G}} + \frac{2\varepsilon\xi}{3}\frac{\dot{X}_{\mathrm{G}}}{\sqrt{\Delta}} + \frac{\kappa\varepsilon\xi}{6}\left[\frac{19}{4}\frac{\dot{\Delta}\dot{X}_{\mathrm{G}}}{\Delta^{7/2}} - \frac{\dot{\Delta}\dot{\Theta}}{\Delta^{7/2}} + \frac{1}{2}\frac{\ddot{\Theta} - \ddot{X}_{\mathrm{G}}}{\Delta^{5/2}}\right] = 0$$

$$\ddot{\Delta} + \xi\frac{\dot{\Delta}}{\Delta^{3/2}} + \frac{\kappa\xi}{4}\left[21\frac{\dot{\Delta}^2}{\Delta^{9/2}} - \frac{(\dot{\Theta} - \dot{X}_{\mathrm{G}})^2}{\Delta^{7/2}} - \frac{15}{2}\frac{\ddot{\Delta}}{\Delta^{7/2}}\right] = 0$$

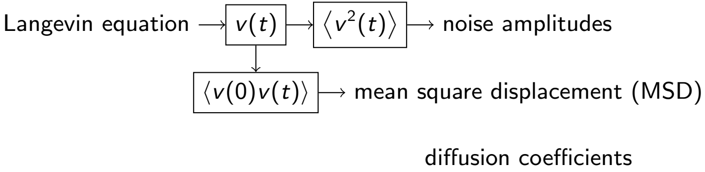
$$\ddot{\Theta} + \frac{4\varepsilon\xi}{3}\frac{\dot{\Theta}}{\sqrt{\Delta}} + \frac{\kappa\varepsilon\xi}{3}\left[\frac{19}{4}\frac{\dot{\Delta}\dot{\Theta}}{\Delta^{7/2}} - \frac{\dot{\Delta}\dot{X}_{\mathrm{G}}}{\Delta^{7/2}} + \frac{1}{2}\frac{\ddot{X}_{\mathrm{G}} - \ddot{\Theta}}{\Delta^{5/2}}\right] = 0$$

T. Salez, and L. Mahadevan, *J. Fluid Mech.* **2015**, *779*, 181-196

⊙ Langevin equation

$$\dot{v} = -\gamma v + \delta F/m$$

P. Langevin, *Compt. Rendus* **1908**, *146*, 530-533.



⊙ Modified fluctuation-dissipation relation

$\dot{v}_i$ coefficients $\longrightarrow$ mass matrix $M_{\alpha\beta}(\kappa, \Delta)$

real mass m_i $\longrightarrow$ effective friction $\gamma_{\text{eff}}(\kappa, \Delta)$

v_i coefficients $\longrightarrow$ friction matrix $\gamma_{\alpha\beta}(\kappa, \Delta)$

$$\boxed{\gamma_{\text{eff}} = M_{\alpha\beta}^{-1} \cdot m_\alpha \cdot \gamma_{\alpha\beta}}$$

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$\dot{v}_i$ coefficients $\longrightarrow$ mass matrix $M_{\alpha\beta}(\kappa, \Delta)$ $\longrightarrow$

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v_i coefficients $\longrightarrow$ friction matrix $\gamma_{\alpha\beta}(\kappa, \Delta)$ $\longrightarrow$

$$\boxed{\gamma_{\text{eff}} = M_{\alpha\beta}^{-1} \cdot m_\alpha \cdot \gamma_{\alpha\beta}}$$

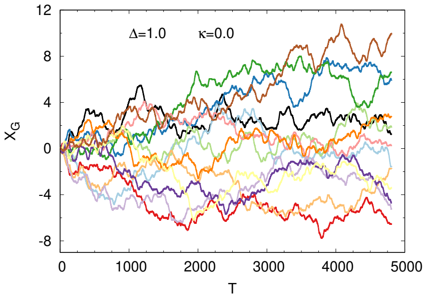
$v_i = v_{i0} + \kappa \cdot v_{i1} \longrightarrow \langle v_{i0} v_{i1} \rangle (t, \kappa)$ $\longrightarrow$

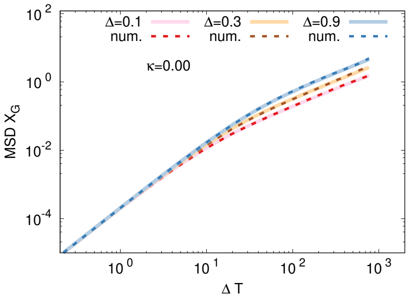
$\gamma_i = \gamma_{i0}(\Delta) + \kappa \cdot \gamma_{i1}(\Delta)$ $\longrightarrow$ noise correlator amplitude

$\delta F_i = \delta F_{i0} + \kappa \cdot \delta F_{i1} \longrightarrow \langle \delta F_{i0} \delta F_{i1} \rangle (\kappa)$ $\longrightarrow$

$$\boxed{\langle \delta F_i(\tau_1) \delta F_i(\tau_2) \rangle \propto 2 k_{\mathrm{B}} T \; m_i \; \gamma_{i0} \; \delta(\tau_1 - \tau_2) \cdot \left[1 - \kappa \cdot \frac{\gamma_{i1}(\Delta)}{\gamma_{i0}(\Delta)} \right]}$$

- Simulation with **fixed** height (Δ) - Effect of a rigid wall ($\kappa = 0$)





$\epsilon = 0.1 \qquad \xi = 1.0 \qquad \Delta \equiv \Delta(0)$

$X_{\rm G}(0) = \dot{X}_{\rm G}(0) = \Theta(0) = \dot{\Theta}(0) = 0$

At long time:

$$\left\langle \Delta X_{\rm G}^2 \right\rangle \propto 2D(\kappa, \Delta)\Delta T$$

$$\log \left\langle \Delta X_{\rm G}^2 \right\rangle = \log \Delta T + \log D(0, \Delta) + C$$

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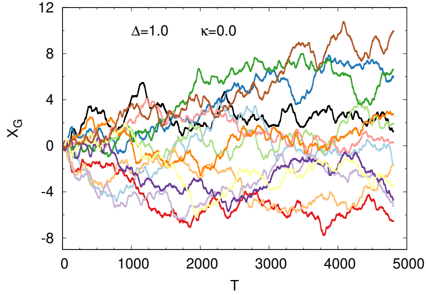

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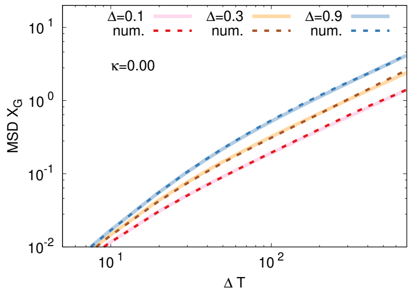

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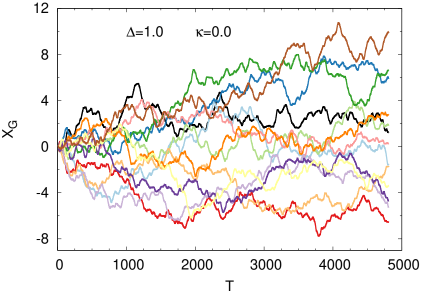

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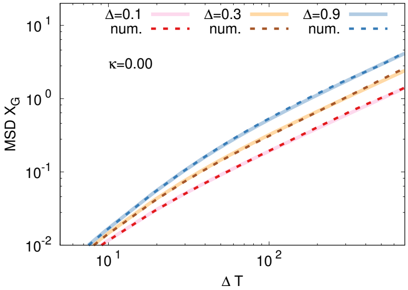

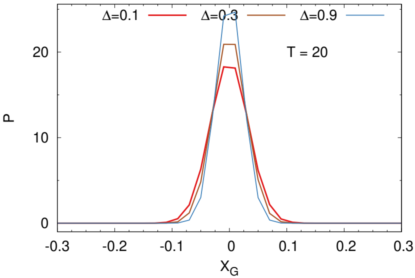

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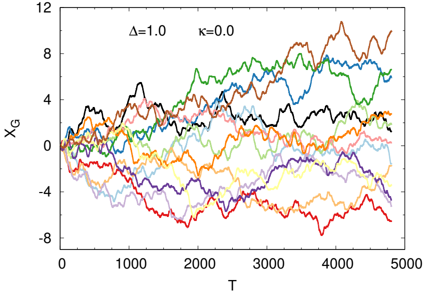

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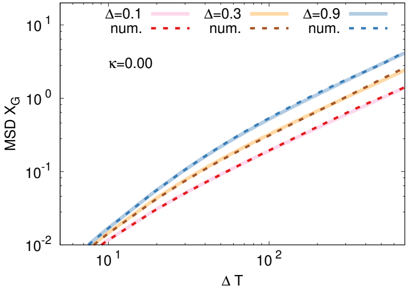

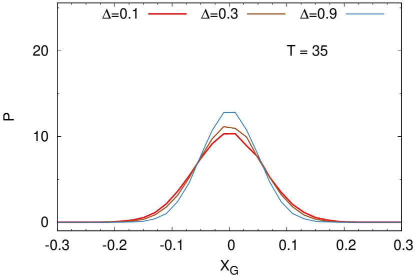

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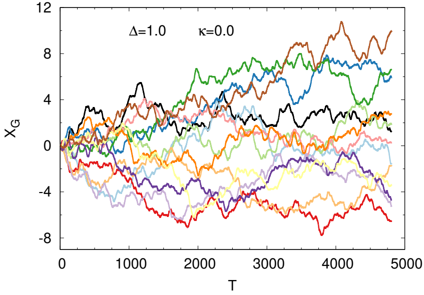

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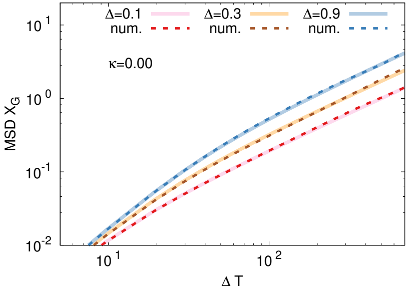

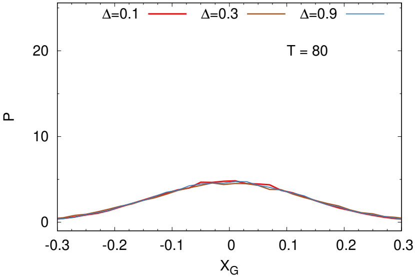

- Simulation with **fixed** height (Δ) - Effect of compliance ($\kappa \neq 0$)

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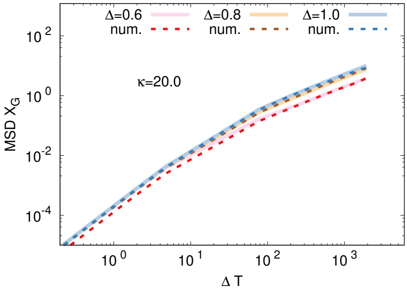
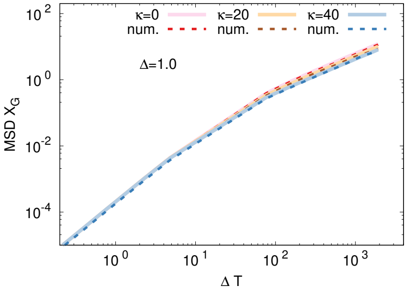
$$\log \left\langle \Delta X_{\mathrm{G}}^2 \right\rangle = \log \Delta T + \log D(\kappa, \Delta) + C$$

$$D(\kappa, \Delta) = D(0, \Delta)\left[1 - \kappa \cdot \frac{\gamma_{i1}(\Delta)}{\gamma_{i0}(\Delta)}\right]$$

$$\log \left\langle \Delta X_{\mathrm{G}}^2 \right\rangle = \log \Delta T + \log D(0, \Delta) + \log \left[1 - \kappa \cdot \frac{\gamma_{i1}(\Delta)}{\gamma_{i0}(\Delta)}\right] + C$$





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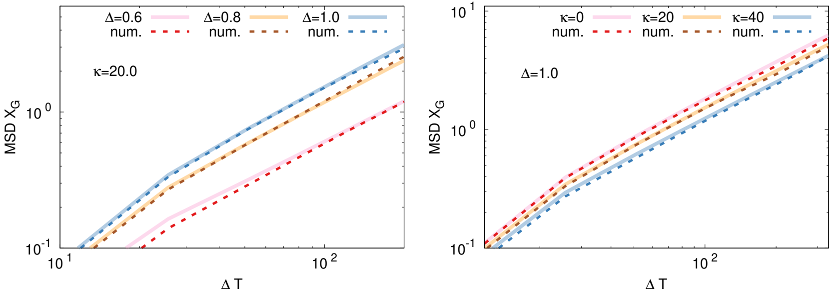
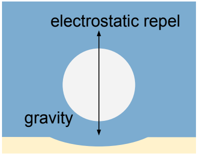
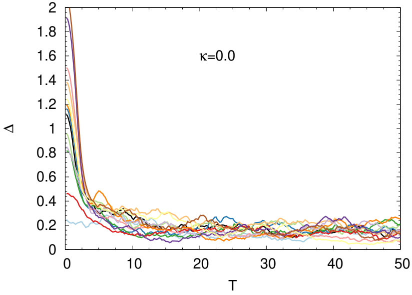


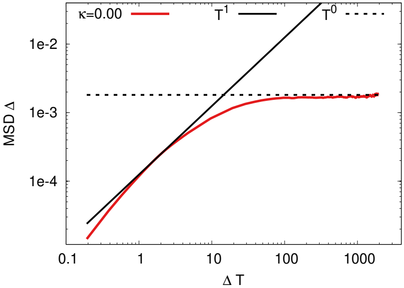
- Simulation with **unfixed** Δ - Electrostatic repel of rigid wall ($\kappa = 0$)

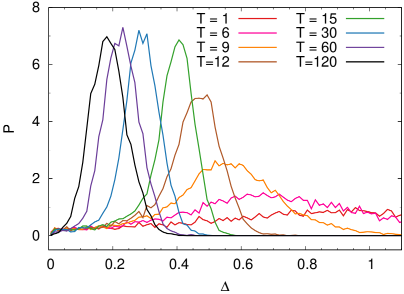


$$\uparrow U_{\mathrm{e}} = k_{\mathrm{B}} T \cdot B \exp\left(-\frac{\Delta}{\ell_D}\right)$$

$$\downarrow U_{\mathrm{g}} = mg\Delta$$







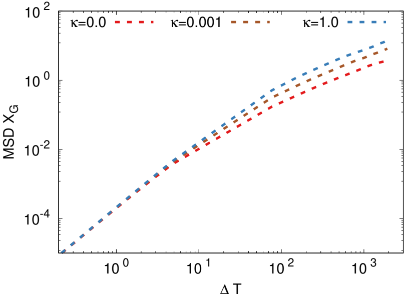
- Simulation with **unfixed** Δ - Effect of compliance ($\kappa \neq 0$)

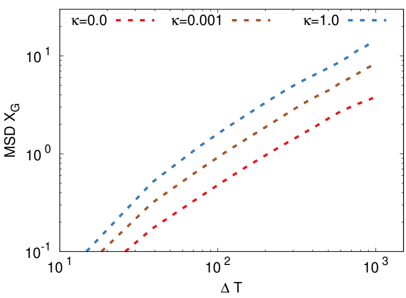
$$\epsilon = 0.1 \quad \xi = 1.0 \quad X(0) = \dot{X}(0) = 0$$

$$\overline{D}(\kappa) = \int_{z_{\min}}^{z_{\max}} P(\Delta)\ D(\kappa, \Delta)\ \mathrm{d}\Delta$$

$$D(\kappa, \Delta) = D(0, \Delta)\left[1 - \kappa \cdot \frac{\gamma_{i1}(\Delta)}{\gamma_{i0}(\Delta)}\right]$$

$$\log \left\langle \Delta X_{\mathrm{G}}^2 \right\rangle = \log \Delta T + \log \overline{D}(\kappa) + C$$





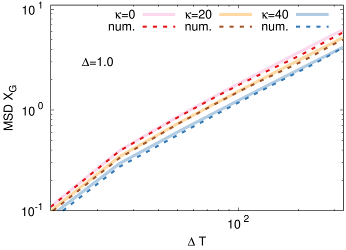
Summary

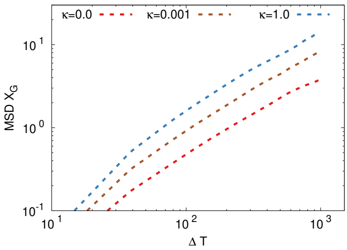
- Noise-correlator amplitude affected by soft surface:

$$\boxed{\langle \delta F_i(\tau_1) \delta F_i(\tau_2) \rangle \propto 2 k_{\mathrm{B}} T \;\, m_i \;\, \delta(\tau_1 - \tau_2) \cdot (\gamma_{i0} - \kappa \cdot \gamma_{i1})}$$

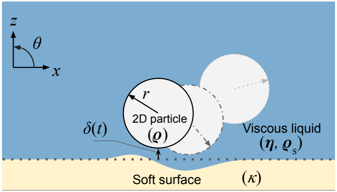
- Less time consumed to enter the diffusive region;
- Diffusion coefficients affected by softer surface;

$$D(\kappa, \Delta) = D(0, \Delta) \left[1 - \kappa \cdot \frac{\gamma_{i1}(\Delta)}{\gamma_{i0}(\Delta)} \right]$$





Perspective



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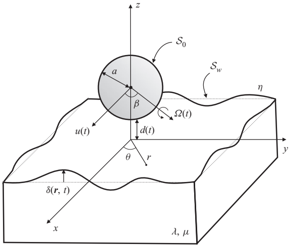

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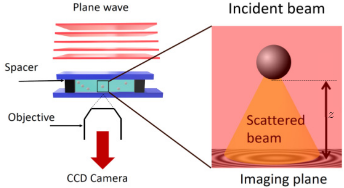

M. Lavaud, et al. *Phys. Rev. Res.* **2021**, *3(3)*, L032011.

- 2D toy model toward the 3D case;
- Experimental verifications;
- "Target finding" diffusion near soft walls;

Acknowledgements

EMetBrown Group @ LOMA, Univ. Bordeaux

Contact:

yilin.ye@ens.psl.eu