Brownian Motion near a Soft Surface

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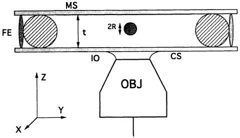
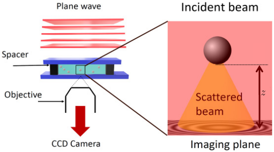
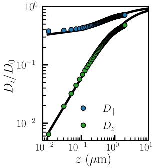
◦ Confined Brownian motion & ElastoHydroDynamic (EHD) lift force

There are lower diffusion coefficients near a rigid surface.







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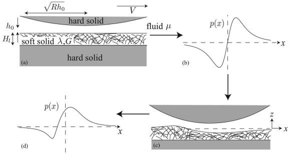
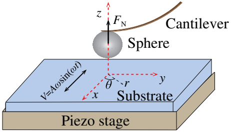
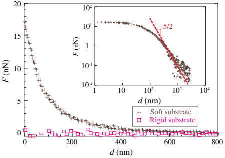
Asymmetric deformation of soft walls leads to a lift force.







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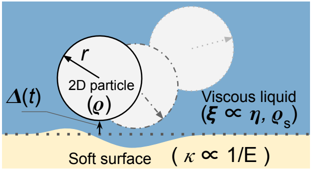
⊙ EHD interactions & Modified fluctuation-dissipation relation

Equations of motion (EOM) are non-linear coupled.



$$\ddot{X}_{\mathrm{G}} + \frac{2\varepsilon\xi}{3}\frac{\dot{X}_{\mathrm{G}}}{\sqrt{\triangle}} + \frac{\kappa\varepsilon\xi}{6}\left[\frac{19}{4}\frac{\dot{\triangle}\dot{X}_{\mathrm{G}}}{\triangle^{7/2}} - \frac{\dot{\triangle}\dot{\Theta}}{\triangle^{7/2}} + \frac{1}{2}\frac{\ddot{\Theta} - \ddot{X}_{\mathrm{G}}}{\triangle^{5/2}}\right] = 0$$

$$\ddot{\triangle} + \xi\frac{\dot{\triangle}}{\triangle^{3/2}} + \frac{\kappa\xi}{4}\left[21\frac{\dot{\triangle}^2}{\triangle^{9/2}} - \frac{(\dot{\Theta} - \dot{X}_{\mathrm{G}})^2}{\triangle^{7/2}} - \frac{15}{2}\frac{\ddot{\triangle}}{\triangle^{7/2}}\right] = 0$$

$$\ddot{\Theta} + \frac{4\varepsilon\xi}{3}\frac{\dot{\Theta}}{\sqrt{\triangle}} + \frac{\kappa\varepsilon\xi}{3}\left[\frac{19}{4}\frac{\dot{\triangle}\dot{\Theta}}{\triangle^{7/2}} - \frac{\dot{\triangle}\dot{X}_{\mathrm{G}}}{\triangle^{7/2}} + \frac{1}{2}\frac{\ddot{X}_{\mathrm{G}} - \ddot{\Theta}}{\triangle^{5/2}}\right] = 0$$

T. Salez, and L. Mahadevan, *J. Fluid Mech.* **2015**, *779*, 181-196

Add random force into EOM for modified fluctuation-dissipation relation.

$$\dot{v} + f(\triangle)\ v + \kappa\ g(\dot{v}, v, \triangle) = 0 \qquad \rightarrow \qquad \dot{v} = -\gamma_{\mathrm{eff}}\ v + \delta F/M$$

$$v_i = v_{i0} + \kappa \cdot v_{i1} \longrightarrow \langle v_{i0} v_{i1} \rangle\,(t, \kappa)$$

$$\gamma_i = \gamma_{i0}(\triangle) + \kappa \cdot \gamma_{i1}(\triangle) \longrightarrow \text{noise correlator amplitude}$$

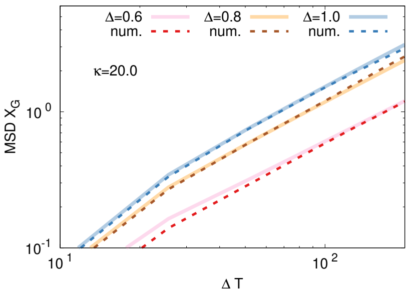
$$\delta F_i = \delta F_{i0} + \kappa \cdot \delta F_{i1} \longrightarrow \langle \delta F_{i0} \delta F_{i1} \rangle\,(\kappa)$$

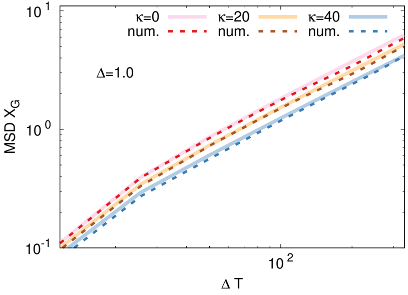
$$\boxed{\langle \delta F_i(\tau_1)\delta F_i(\tau_2)\rangle \propto 2k_{\mathrm{B}}T\ m_i\ \gamma_{i0}\ \delta(\tau_1 - \tau_2)\cdot\left[1 - \kappa\cdot\frac{\gamma_{i1}(\triangle)}{\gamma_{i0}(\triangle)}\right]}$$

- Simulation with **fixed** height (Δ) - Effect of compliance ($\kappa \neq 0$)

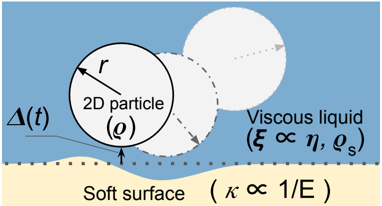
Diffusion coefficient depends on vertical positions and soft wall modulus.

$$D(\kappa, \Delta) = D(0, \Delta)\left[1 - \kappa \cdot \frac{\gamma_{i1}(\Delta)}{\gamma_{i0}(\Delta)}\right]$$





◇ Take-home messages



- **Noise-correlator amplitude affected by soft surface:**

$$\left\langle \delta F^2 \right\rangle \propto \left[1 - \kappa \cdot \frac{\gamma_{i1}(\Delta)}{\gamma_{i0}(\Delta)} \right]$$

- **Less time consumed to enter the diffusive region;**
- **Diffusion coefficients affected by softer surface;**

$$D(\kappa, \Delta) = D(0, \Delta) \left[1 - \kappa \cdot \frac{\gamma_{i1}(\Delta)}{\gamma_{i0}(\Delta)} \right]$$